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NOTE

Eliminating Boundary-Induced Errors in the Pseudo-Current Method

The pseudo-current algorithm [1] provides a method for controlling errors in Gauss' law in electromagnetic particle-in-cell codes. It does so by adding to Ampere's law a multiple of the gradient of Gauss' law:

$$\partial E/\partial t = \text{curl } B - J + d \text{ grad } F, \tag{1}$$

where

$$F = \text{div } E - \rho. \tag{2}$$

The equations are written in dimensionless form. F satisfies a diffusion equation with a source term arising from numerical error in the physically vanishing expression

$$\partial \rho/\partial t + \text{div } J. \tag{3}$$

One advantage of this method is that the charge density, ρ, and the current density, J, can be determined from the particle distribution by any suitable method, not necessarily one in which (3) is required to vanish identically. It is equally valid in any number of dimensions and in any coordinate system. Of course, when (3) is not identically zero because of numerical inaccuracies, Gauss' law will not be satisfied and the quantity F will grow. For any reasonable algorithm its rate of growth will be small. The pseudo-current algorithm ensures that F remains small for all time.

It has been found that boundaries at which particles are either emitted or absorbed can produce spurious errors with this method unless special care is taken. We shall describe the source of these errors and present a simple method for eliminating them. Referring to Fig. 1, suppose that the surface $L-1$ is a physical or numerical boundary. In a typical linear weighting scheme, a volume enclosing one full cell on all sides is used to find quantities defined at the center. For the current density near a boundary, however, only half a cell is available on one side. Furthermore, particles abruptly appear and vanish in this truncated volume as they are emitted or absorbed. Thus, determining the current density at the first half cell into the domain requires special treatment. The simplest method is simply to use a weight factor to compensate for the missing volume. When this is done, though, the current density at the first half cell, labeled $JX(L, M)$ in the figure, is not as accurately determined as current densities computed in the interior. This inaccuracy in J can result in enhanced generation of the quantity "F" near boundaries. It can, in fact, become the dominant source of this error term. Some of this newly generated F will diffuse back to the boundary and vanish from the problem, but some diffuses into the interior of the domain.

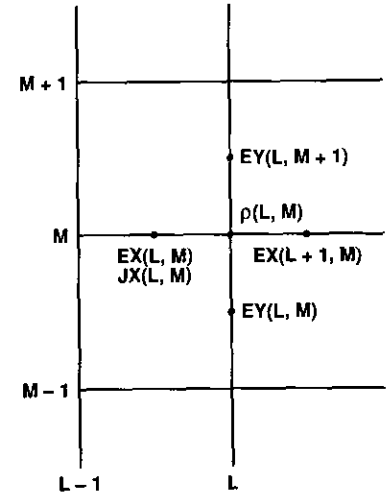

FIG. 1. Centering of the electric field, current density, and charge density near an emitting or absorbing boundary ($L-1$).

This non-physical contamination can be easily prevented by using the following predictor-corrector method. The electric fields are determined as usual from Ampere's law and the particles are moved accordingly. When the new density is obtained, the electric fields half a cell in from all boundaries, represented in the figure by $EX(L, M)$, are recomputed from Gauss' law by using the charge density, $\rho(L, M)$, and the three other interior field components. With this new electric field, "F" vanishes (that is, Gauss' law is satisfied) not only on boundaries, where it is defined to be zero, but on the first cell in from the boundaries as well. Since most of the time in these codes is used moving particles, the addition of this field solution on a lower dimensional surface requires negligible time but can significantly reduce errors in Gauss' law throughout the domain.

REFERENCE

1. B. M. Marder, *J. Comput. Phys.* **68**, 48 (1987).

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